NEW YORK UNIVERSITY, APPLIED MATHEMATICS GROUP

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Dr. T. von Karman
AAF Scientific Advisory Board
Headquarters Army Air Forces
Room 4D-1070
Pentagon Building
Washington 25, D.C.

Dear v. Karman:

Since I returned from visiting you all calculations for the airfoil in a flow with Mach number 1.6 were redone. Some numerical mistakes were found. I am rather sure that the new results are correct; they are shown on the enclosed graphs. These results are compared with first and second approximations obtained by expanding the problems in powers of the thickness ratio A/L^2. These approximations agree with those given by Ackeret and Busemann.

It is interesting to note that the drag is quadratic and the pressures at both tips are linear in A/L^2 and are not improved by the second approximation; the second order improvement of the airfoil shape would therefore effect the drag only in third approximation.

The drag that we have calculated turns out less than the second approximation, as is expected, but the deviation is very small. I wonder whether it pays to have these laborious calculations carried through for other Mach numbers.

Another mistake that was found resulted in some difficulties of a general nature. Consequently the whole theory is not yet in a complete state.

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As I had explained to you, the calculated profiles are not exactly those that give the minimum drag. A minimizing sequence for a fixed initial angle will either approach a profile that begins with a straight segment (Case I) or one whose initial angle is less than the prescribed one (Case III). We have picked an initial angle for which the minimum belongs to Case II, in which the initial angle is the prescribed one but changes immediately. I still believe that it gives a good approximation to do so; the drag is indeed decreased as our graphs show. Earlier I had claimed that the minimum on varying the initial angle belongs to Case I; it now turns out that it belongs to Case III. In other words, a profile for which the drag is near its lower limit has a strong curvature near the tip. The assumption that the flow around the profile could be described as a simple wave and that the waves reflect from the shock can be forgotten, is then no longer justified.

If this assumption were still justified one would arrive at the paradoxical conclusion that a minimum drag would be obtained by making the entropy as large as possible. Indeed, since the stagnation pressure decreases with increasing entropy, the same is true for the pressure at any point of the Mach line with a given angle of flow direction in a simple wave. To achieve minimum drag one would therefore have to strengthen the shock at the tip, perhaps even to round off the tip of the nose to create a normal shock there standing a little bit ahead of the tip.

Another argument is this: Suppose one modifies the shape of the profile slightly only at the tip by changing the angle and suppose one considers the sequence of profiles obtained by letting this modification approach zero in such a way that

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the modified initial angle is kept fixed. Then clearly the flow as a whole will not differ very much from that around the unmodified profile.

Nevertheless, the entropy along the contour is permanently different. Consequently there is a boundary layer along the contour. The same arguments that are used for the viscosity boundary layer theory yield also here that the pressure should be essentially constant across theboundary layer. That, however, is in contradiction to the assumption that the flow can be described as a simple wave. The adjustment of the pressure to that of the main stream must be accomplished by reflected and re-reflected waves.

For a treatment of the problem including the waves reflected from the shock, a third approximation seems reasonable. As a matter of fact, the reflected wave drops out in first or second approximation. That this is so does not seem obvious to me, because the well-known fact that the entropy changes only in third order is no sufficient reason. For a justification one need show that the relation between pressure and flow angle across a shock and across a simple compression wave (a Meyer wave run through backwards) disagree only in third order. I have verified this fact by calculation; but I do not have a short-cut reason for it.

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Since the second approximation is already very accurate in the range considered, it seems more reasonable to develop a third order approximation with reflected waves taken into account, rather than an exact treatment without doing so.

The entropy in third order turns out to be constant and the flow is consequently still a potential flow in third order. The influence of entropy change and reflected wave on the pressure distribution along the profile shows up in that the pressure at a point of the profile depends in third order not only on the slope at that point but also on the slope at the tip. The term expressing this dependence drops, however, out from the expression for the drag. Thus the problem of minimizing the drag is still reasonable in third order and it would at least be necessary to go to the fourth order to decide whether minimum drag requires any irregularity at the tip. To carry this out would be very laborious and therefore I am sending the enclosed graphs as they are. (I had delayed doing so for two weeks because I had hoped I could settle the question in the meantime.)

With best regards,

Very truly yours,

K. O. Friedrichs

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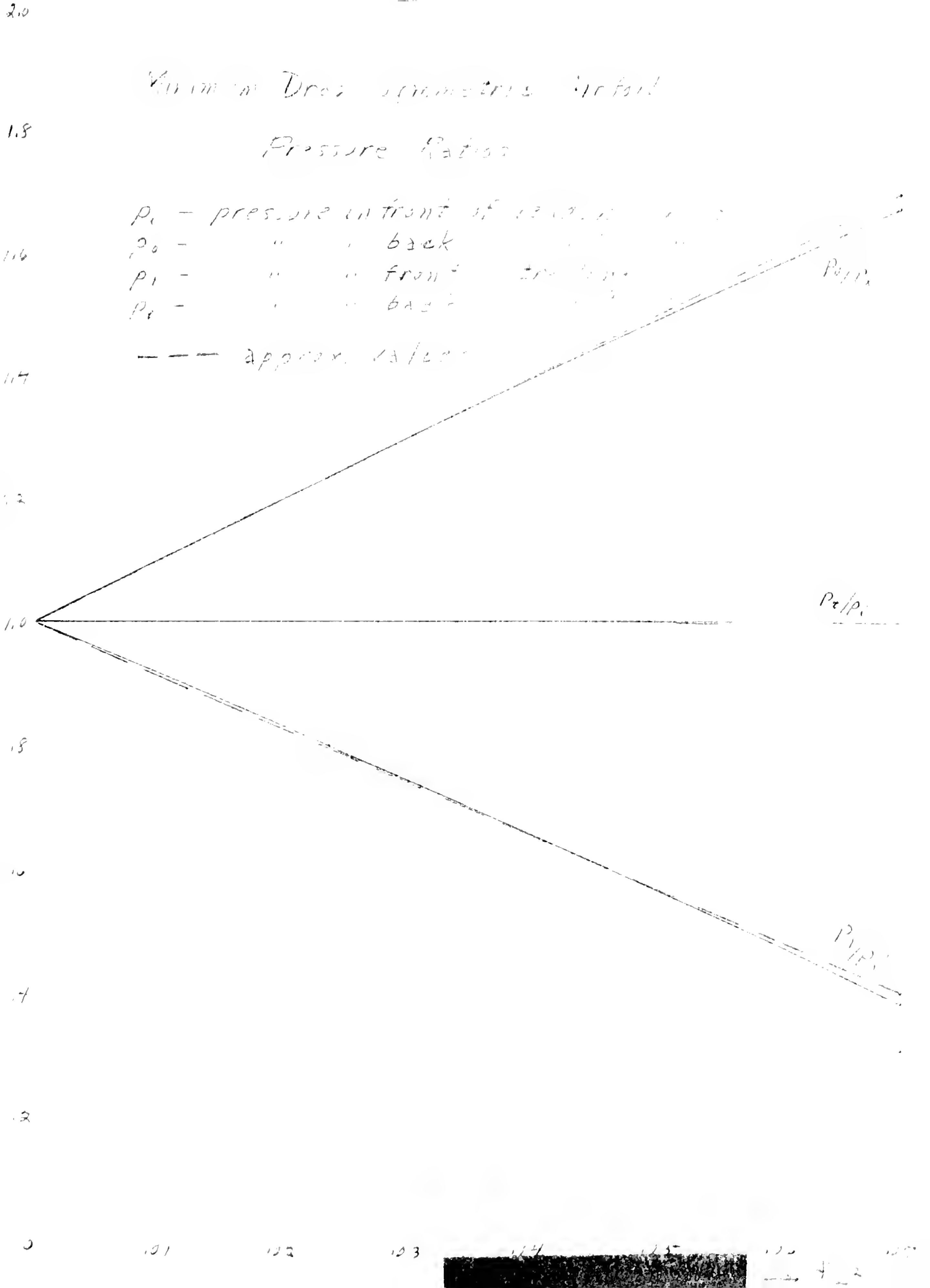

2.0
1.8
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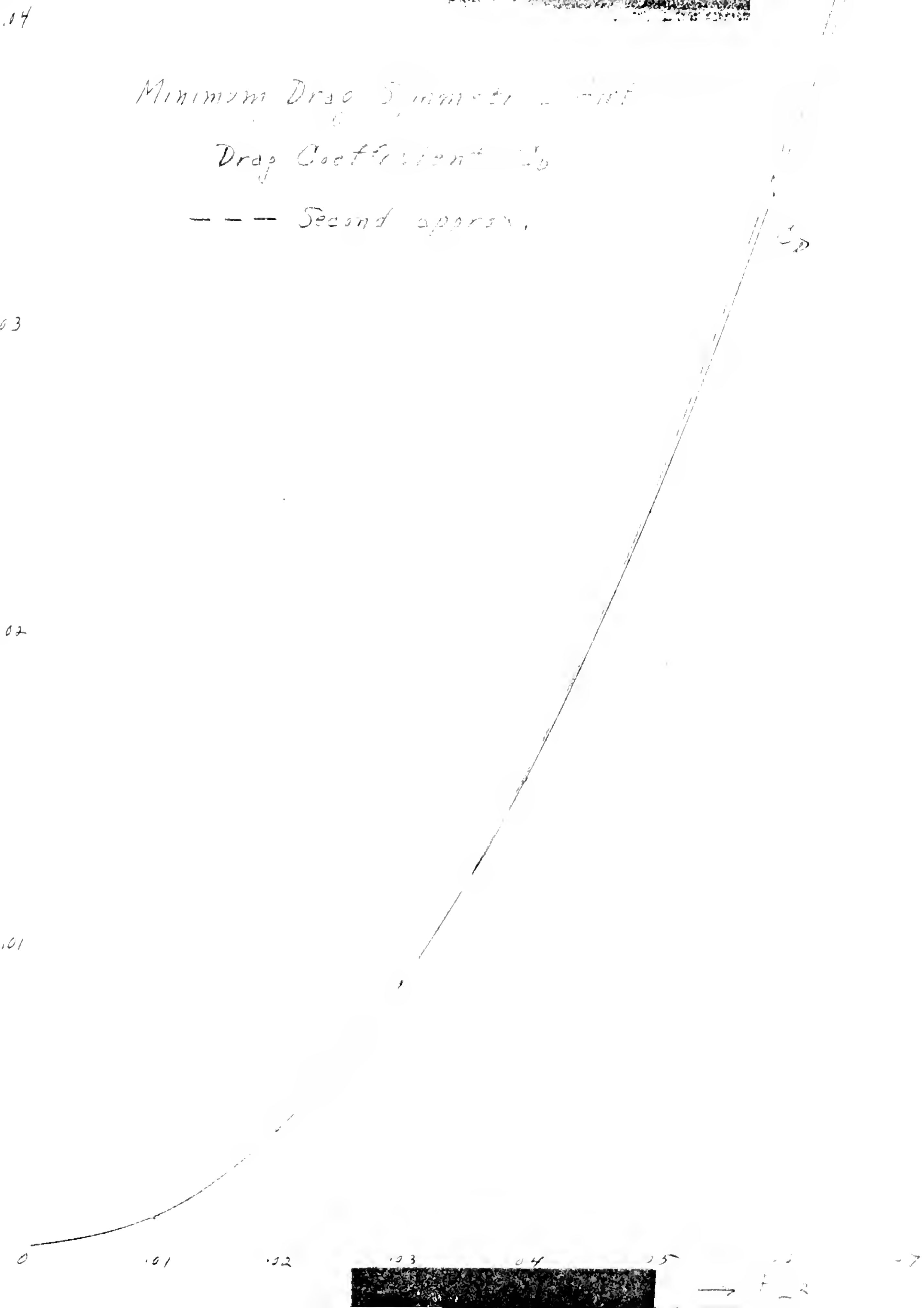
back
front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Minimum Drag
Drag Coefficient
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.03
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0
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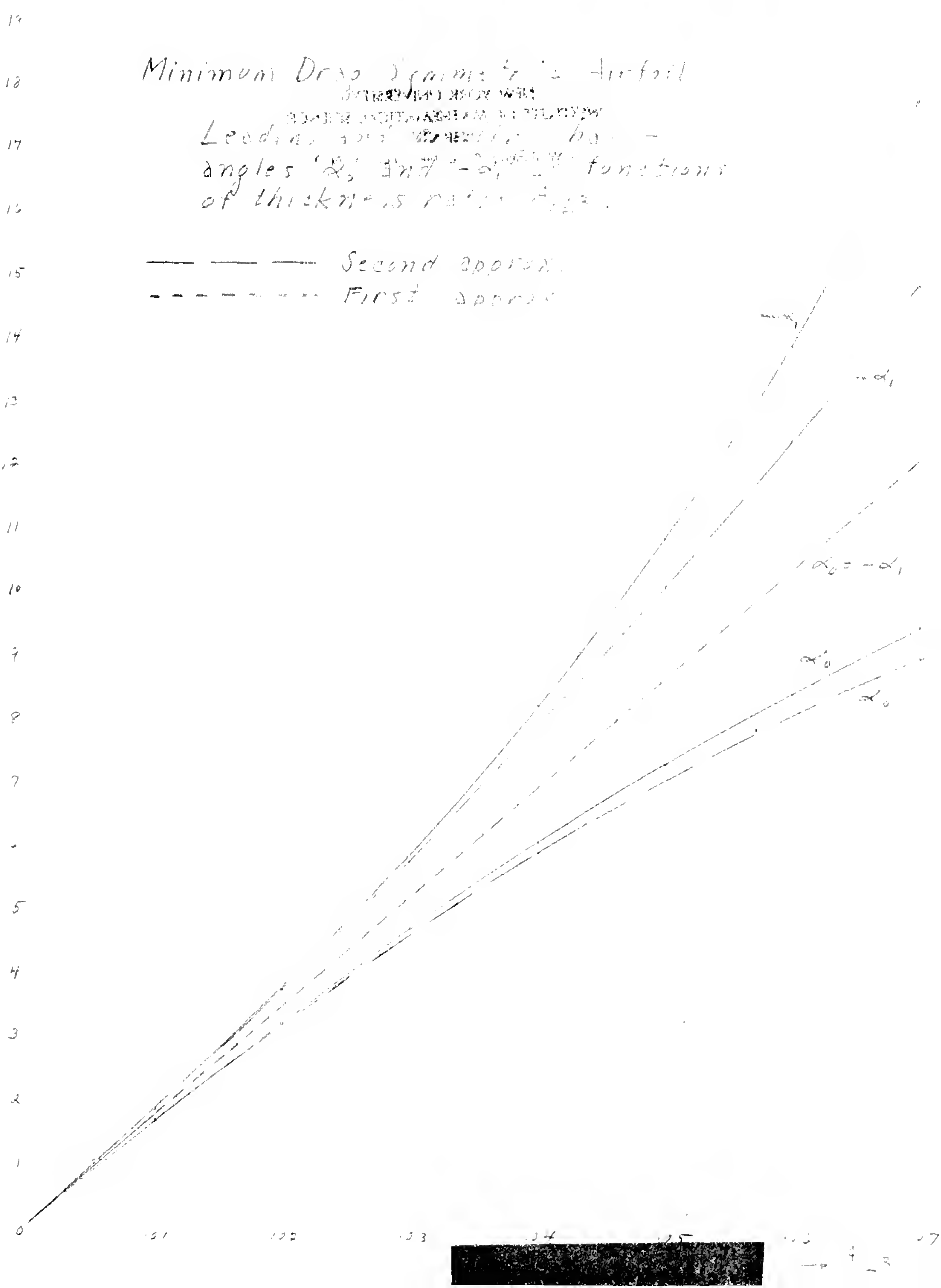
Minimum Drag Symmetric Airfoil
Leading and trailing edge
angles α_0 and $-\alpha_1$ as functions
of thickness ratio
Second approx.
First approx.
$-\alpha_1$
$-\alpha_1$
$\alpha_0 = -\alpha_1$
α_0
α_0
0
1
2
3
4
5
7
8
9
10
11
14
15
17
.01
.02
.03
.04
.05
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